Sequential-Move Games with Information Costs

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Abstract

I study finite two player normal form games where player 2 (the 'follower') can observe the move of player 1 (the 'leader') by paying a small cost. I characterize the limit set of perfect equilibria of this game as the cost of information converges to zero and provide a simple algorithm for constructing it. Limit equilibria have the following properties: (a) both players choose pure strategies; (b) the follower plays a best response; (c) even though the set of limit equilibria always contains the Stackelberg equilibrium it can contain strategy profiles which are not even Nash equilibria of the normal form game. In fact, the follower will only purchase information in the Stackelberg equilibrium if the Stackelberg equilibrium is not a Nash equilibrium. Similar to Yariv and Solan (2004), the subgame perfect solution concept is therefore not robust to the introduction of small information costs. The Stackelberg equilibrium only reemerges as the unique limit equilibrium if we allow for the possibility that the leader is irrational.

I test the theory experimentally using the classic Battle of the Sexes game with information acquisition. I find that subjects coordinate on the Stackelberg equilibrium *and* purchase information which can only be reconciled with theoretical predictions if it is common knowledge amongst subjects that the leader can be irrational.

1 Introduction

I study finite two player normal form games where player 2 (the 'follower') can observe the move of player 1 (the 'leader') by paying a small cost. I characterize the limit set of perfect equilibria of this game as the cost of information converges to zero. Limit equilibria have the following properties: (a) both players choose pure strategies; (b) the follower plays a best response; (c) even though the set of limit equilibria always contains the Stackelberg equilibrium it can contain strategy profiles which are not even Nash equilibria of the normal form game. In fact, the follower will only purchase information in the Stackelberg equilibrium if the Stackelberg equilibrium is not a Nash equilibrium. Similar to Yariv and Solan (2004), the subgame perfect solution concept is therefore not robust to the introduction of small information costs. The Stackelberg equilibrium only reemerges as the unique limit equilibrium if we allow for the possibility that the leader is irrational.

The main theorem 2 provides a simple condition for checking whether a strategy profile is in the limit set and whether the follower purchases information with positive probability. For example, in the two-player, three by three game shown in figure 1 the limit set supports both the Stackelberg equilibrium (T,L) and the profile (B,R) which is not even a Nash equilibrium.[1]

Intuitively, even a small cost of information provides the follower with a credible commitment to ignore the leader's choice of strategy which undermines the usual Stackelberg argument. The follower will only purchase costly information if she can learn something useful from it. This implies that the leader has to have an incentive to deviate to a more profitable strategy. Therefore, the Stackelberg equilibrium in the classic Battle of the Sexes game, for example, is supported in the limit set only when the follower purchases no information in equilibrium. In contrast, coordination on the follower's preferred outcome is also in the limit set and the follower purchases information with positive probability.

The Battle of the Sexes game in fact provides a simple experimental test of the theory. 346 subjects played a repeated Battle of the Sexes game with random rematching and I find that (a) subjects learn to coordinate on the leader's preferred action over time and (b) the propensity of the follower to buy information increases over time. These findings suggest that

[1]Under (T,L) the follower buys information with probability $\frac{2}{5}$ and under (B,R) the follower buys information with probability $\frac{1}{3}$.

	L	C	R
T	2,3	4,2	−3,1
M	4,0	−1,3	2,2
B	−3,1	1,2	1,3

Figure 1: Both (T,L) and (B,R) are supported in the limit set.

forces such as possible irrationality of the leader push players back towards the Stackelberg equilibrium and suggest that subgame perfect reasoning might be more robust than theory predicts.

The theory part of this paper is closely related to a recent paper by Yariv and Solan (2004) who were the first to study normal-form games with espionage. The information device in my paper is more primitive than the class of information devices studied by Yariv and Solan (2004) where the follower selects a device from a convex set and pays according to a continuous cost function. In contrast, the follower in my model can only choose between two devices: no information at zero cost and perfect information at some positive cost.[2]. While my setup does not lend itself to study noisy information acquisition it has the advantage that all limit equilibria are in pure strategies and there is a simple algorithm to check for limit equilibria.

The paper is organized as follows. Section 2 introduces the notion of a limit equilibrium and develops the theory leading to the main result of theorem 2. Section 3 describes the experimental results.

2 Theory

The base game is a normal form game with two players. Player 1 has n_1 strategies in his strategy set $S_1 = \{a_1, .., a_{n_1}\}$ and player 2 can choose n_2 strategies from his strategy set

[2]Because the set of information devices is non-convex the model in this paper is not a special case of Yariv and Solan (2004)

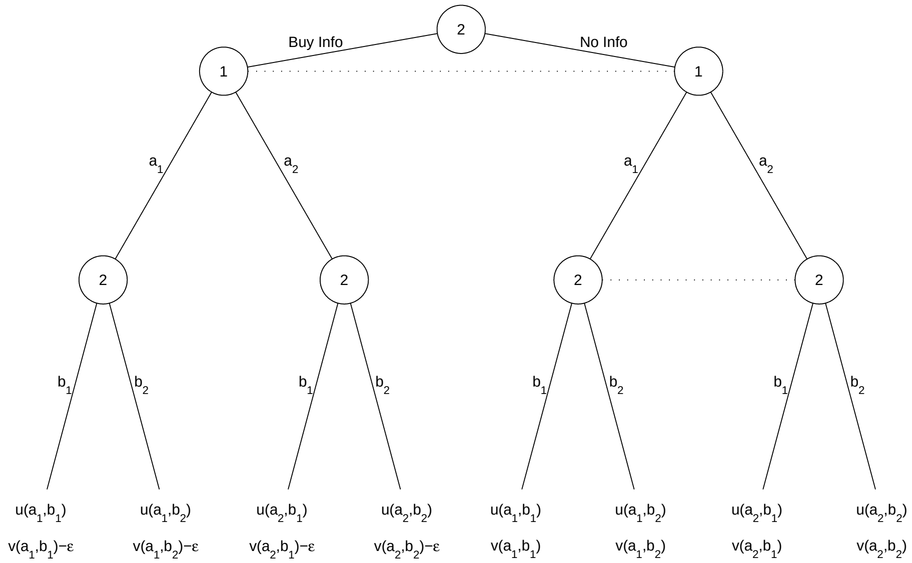

Figure 2: Extensive form representation of sequential two by two game

$S_2 = \{b_1, .., b_{n_2}\}$. Player 1's utility is described by a function $u : S_1 \times S_2 \to \Re$ and player 2's utility by a function $v : S_1 \times S_2 \to \Re$. I make a number of technical assumptions: (1) there are no strictly dominated strategies[3]; (2) the follower has a unique best response to any pure strategy; (3) the leader is never indifferent between two strategies for any pure strategy played by the follower; (4) there is a unique Stackelberg equilibrium of the sequential game.[4] The set of pure strategy Nash equilibria of the normal form game is denoted with Γ_P.

I next look at the sequential version of the above normal form game where player 1 moves first and player 2 can buy information at cost ϵ. The extensive-form representation of a sequential two by two game is given as an example in figure 2. For simplicity I call an sequential game with information cost ϵ an ϵ-game. The special case where the information cost $\epsilon = 0$ is called the *no-cost game*. I define the constant m as the minimum utility difference between the first and second-best response of the follower to any of the leader's actions which by assumption (2) is strictly positive.

I am studying trembling-hand perfect equilibria Selten (1975) of the extensive form game. Since each ϵ-game is finite and has perfect recall it has at least one such equilibrium. In

[3]Strictly dominated strategies are never played in the sequential game with information acquisition costs.
[4]Assumptions (2) to (4) can be relaxed but they simplify the exposition.

finite games trembling-hand perfection is a refinement of Perfect Bayesian equilibrium. It is a particularly convenient refinement because it ensures that player 2 will always buy information if it is costless. I denote the set of trembling-hand perfect equilibria of the ϵ-game with Γ_ϵ. The equilibrium limit set Γ^* consists of all limit points of all sequences (σ_{ϵ_n}) where $\sigma_{\epsilon_n} \in \Gamma_{\epsilon_n}$ and $\epsilon_n \to 0$ as $n \to \infty$.

Proposition 1 *The equilibrium limit set Γ^* is non-empty.*

Proof: See appendix A.1.

Intuitively, a limit equilibrium describes approximate play as the cost of information declines. In the remainder of this section I will compare the equilibrium limit set Γ^* to the set of trembling-hand perfect equilibria of the no-cost game which is just a singleton:

Proposition 2 *In the no-cost game the set Γ_0 of trembling hand perfect equilibria consists of the Stackelberg equilibrium σ_S of the sequential game and player 2 buys information for sure.*

Proof: See appendix A.2.

2.1 Pure Strategy Equilibria of ϵ-Games

I next analyze in detail the equilibrium limit set of the ϵ-game. I will show that the Stackelberg equilibria are still part of the limit set but that the limit set is typically strictly larger.

I start with characterizing the pure-strategy equilibria. It is convenient to construct a set $\tilde{\Gamma}_P$ of pure-strategy profiles in the extensive form game from the set Γ_P of pure Nash equilibria of the base game. For each $(s_1, s_2) \in \Gamma_P$ I define a corresponding strategy profile in the ϵ-game where player 1 plays s_1, and player 2 buys no information and plays s_2 on the equilibrium path and his unique response off the equilibrium path.

Proposition 3 *The set $\tilde{\Gamma}_P$ contains exactly the pure-strategy equilibria of the ϵ-game for any $\epsilon > 0$. Furthermore, it is subset of the limit set Γ^*.*

Proof: See appendix A.3.

Intuitively, even an arbitrarily small information cost gives commitment power to player 2 not to react to deviations by player 1. I refer to this as the *commitment effect* which was first discussed in Yariv and Solan (2004). Hence the Nash equilibria of the base game are also equilibria in the ϵ-game.

2.2 Mixed Equilibria of ϵ-Games

Mixed equilibria of the ϵ-game are more interesting than pure equilibria because they give rise to elements in the limit set where the follower buys information with positive probability. Note, that in any mixed equilibrium player 1 cannot play a pure strategy - otherwise player 2 would not buy information and play his best response instead.

Can there be mixed equilibria where player 2 does *not* buy information? The next result shows that player 2 will always buy information with strictly positive probability in a mixed equilibrium provided the cost of information is sufficiently small.

Proposition 4 *In any mixed trembling-hand perfect equilibrium of the ϵ-game with information cost $0 < \epsilon \leq \frac{m}{2}$ player 2 buys information with strictly positive probability.*

Proof: See appendix A.4.

However, player 2 also cannot buy information for sure in any equilibrium.

Lemma 1 *There is no trembling-hand perfect equilibrium of the ϵ-game where player 2 buys information for sure.*

Proof: See appendix A.5.

Combining the previous two results we now know that in all mixed equilibria of the sequential game with sufficiently small but positive cost of information player 2 buys information with probability $0 < \beta < 1$ - hence she has to be indifferent between buying information or not. This observation can be used to show that in all mixed equilibria both players play 'almost pure' strategies for small information costs.

Theorem 1 *There are constants $0 < \underline{\beta}, \overline{\beta} < 1$ and $\epsilon^* > 0$ such that for any ϵ-game with $\epsilon < \epsilon^*$ and any mixed trembling-hand perfect equilibrium of the game player 1 plays some strategy a* *with probability of at least* $1 - n_1 \frac{\epsilon}{m}$ *and player 2 plays the best response* $b = BR_2(a)$ *to* a *and buys information with probability* $\underline{\beta} < \beta < \overline{\beta}$.

Proof: See appendix A.6.

Theorem 1 allows me to characterize the missing elements of the equilibrium limit set $\Gamma^* \backslash \tilde{\Gamma}_P$ which are the limit points of sequences of mixed strategy equilibria.

Lemma 2 *The equilibrium limit set* $\Gamma^* = \tilde{\Gamma}_P \oplus \Gamma^*_M$ *consists of the set of pure equilibria* $\tilde{\Gamma}_P$ *and the limit points of sequences of mixed equilibria* Γ^*_M. *These limit points have the following structure: (A) player 1 plays a pure strategy* a*; (B) player 2 buys information with probability* $\underline{\beta} < \beta < \overline{\beta}$*; (C) player 2 plays* $b = BR_2(a)$ *if she does not buy information; (D) player 2 plays her unique best response if she buys information.*

Proof: See appendix A.7.

Note that there can be at most n_1 pairs $(a, BR(a))$ supported in Γ^*_M. There is a simple algorithm to check each such pair. For each strategy $a \in S_1$ define the sets $A_L(a), A_H(a) \subset S_1$ as follows:

$$
\begin{aligned}
A_L(a) &= \{a' \in S_1 | BR_2(a') \neq BR_2(a) \quad \text{and } u(a', b) < u(a, b) < u(a', BR(a')) \quad \} \\
A_H(a) &= \{a' \in S_1 | BR_2(a') \neq BR_2(a) \quad \text{and } u(a', BR(a')) < u(a, b) < u(a', b) \quad \}
\end{aligned}
$$

Also define:

$$
\beta_L(a) = \begin{cases} \max_{a' \in A_L(a)} \frac{u(a,b) - u(a', BR_2(a'))}{u(a',b) - u(a', BR_2(a'))} & \text{if } A_L(a) \neq \emptyset \\ 0 & \text{otherwise} \end{cases} \tag{1}
$$

$$
\beta_H(a) = \begin{cases} \min_{a' \in A_L(a)} \frac{u(a,b) - u(a', BR_2(a'))}{u(a',b) - u(a', BR_2(a'))} & \text{if } A_H(a) \neq \emptyset \\ 1 & \text{otherwise} \end{cases}
$$

Theorem 2 *There is an element in the limit set* Γ^*_M *which supports player 1 playing* a *and player 2 playing* $BR(a)$ *if an only if* $A_L(a)$ *and* $A_H(a)$ *are not both empty and* $\beta_L \leq \beta_H$.

Proof: See appendix A.8.

Theorem 2 can be easily applied to each of the n_1 possible strategy profiles $(a, BR(a))$.

For example, we can now revisit the simple normal form game of figure 1 which was provided as a motivating example in the introduction. The normal form game has no pure strategy equilibria and hence $\tilde{\Gamma}_P = \emptyset$. There are possible strategy profiles which are supported in Γ^*_M: the Stackelberg equilibrium (T, L) and (M, C) and (B, R). Applying theorem 2 we find that (T, L) is supported because $\beta_L = 0$ and $\beta_H = \frac{3}{5} > \beta_L$. Similarly, (B, R) is supported because $\beta_L = \frac{1}{5} < \frac{2}{3} = \beta_H$. However, (M, C) is not supported. Therefore, the limit set supports the Stackelberg equilibrium as well as (B, R).

2.3 Stackelberg Equilibria

I have shown that the limit set Γ^* can contain more than one element while the set of trembling-hand perfect equilibria of the no-cost game is a singleton, namely the Stackelberg equilibrium.

One can show that the Stackelberg action pair is at least always supported in the limit set Γ^*.

Proposition 5 *The Stackelberg strategy profile σ_S is supported in the limit set Γ^*.*

Proof: See appendix A.9.

However, even though the Stackelberg profile is supported in the limit set it is not necessarily in Γ^*_M. This matters because the elements in Γ^*_M have player 2 buy information with positive probability - one might therefore find these limit equilibria more convincing as a prediction of play in ϵ-games.

Corollary 1 *The Stackelberg equilibrium σ_S is an element of Γ^*_M if and only if it is not a Nash equilibrium.*

Proof: This follows immediately from applying theorem 2.

Therefore, the Stackelberg equilibrium will only be supported in the limit set by a point where the follower buys information with positive probability if the leader enjoys a first-mover advantage.

2.4 Leader Irrationality

Now assume that there is a small probability η that the leader is irrational and randomizes equally across all his strategies.

Proposition 6 *If it is common knowledge that the leader is irrational with probability η the equilibrium limit set Γ^* shrinks to the Stackelberg outcome where the follower buys information for sure.*

The proof is obvious: if the leader can play any strategy with probability bounded away from zero the follower will always buy information if it is sufficiently cheap. But the rational leader will anticipate this and play his preferred Stackelberg strategy for sure. I refer to this effect as the *irrationality effect*.

3 Experimental Evidence

It is an empirical question whether the irrationality effect is strong enough to overcome the commitment effect in sequential games with small acquisition costs. The following experiment is designed to address this issue.

One principal difficulty is that the Stackelberg outcome is always in the limit set. Therefore observing coordination on the Stackelberg outcome per se is not inconsistent with either commitment or irrationality effect. However, in the absence of leader irrationality theory predicts that the follower does not purchase information if the Stackelberg equilibrium is also a Nash equilibrium in the normal form game.

This is the case in the simple Battle of the Sexes game as shown in figure 3. This base game has the two pure Nash equilibria (A, A) and (B, B) and (A, A) is also its unique Stackelberg equilibrium. It is easy to check that the set Γ^*_M only supports the non-Stackelberg outcome (B, B) where the follower buys information with probability $\beta = \frac{1}{2}$.

3.1 Design and Empirical Strategy

During the experiment I invite groups of 10 subjects to the lab and rematch them pairwise 5 times to play a sequential Battle of the Sexes game where player 1 is the leader and player

	A	B
A	500,250	0,0
B	0,0	250,500

Figure 3: Battle of the Sexes game

2 is the follower.[5] The roles of players are kept constant throughout the experiment - thus a subjects who plays the role of player 1 will always be matched with a player 2.

There are three treatments - C25, C5 and NI:

C25: Subjects can buy information at cost 25 (11 sessions).

C5: Subjects can buy information at cost 5 (14 sessions).

NI: Subjects cannot buy information (10 sessions).

The last no-information treatment controls for move-order effects. Note that the extensive form game in the no-information treatment is formally equivalent to a simultaneous-move game. However, empirically framing a simultaneous move game as a sequential game can potentially induce a greater degree of coordination on the leader's preferred action in the Battle of the Sexes game either due to timing of moves or labeling of players (see Kreps (1990) and Weber, Camerer, and Knez (2004)). Indeed, Cooper, DeJong, Forsythe, and Ross (1993) provide experimental evidence in support of coordination on the leader's preferred outcome in the sequential game. Therefore, treatment NI provides a useful benchmark to measure the importance of sequential move order in coordination. In contrast, the ability to purchase information creates an equivalent of an outside option for player 2. Cooper, DeJong, Forsythe, and Ross (1993) find that presenting a player with an outside option makes coordination on that player's preferred outcome more likely. Therefore, on one hand, experimental evidence

[5]Only in two sessions do we match players only 3 times. Moreover, in 2 sessions we only had 8 instead of 10 players in the group. We took great care to describe the game in neutral language (see appendices A to D).

points to the likelihood of coordination on row player's preferred choice due to move order effects and on the other hand, ability to purchase information makes coordination on column player's preferred outcome more likely.

3.2 Subjects

The experiment was conducted at a computer lab at Tucuman University, Tucuman, Argentina between October 2002 and April 2003. Subjects were recruited via posters located at central locations on three different campuses. They were promised 12 Argentinian pesos (about US$3.50 at the time) in participation fees plus earnings from the experiment. Subjects were assigned to different sessions and special precautions were taken to ensure that they did not know each other prior to the experiment and did not communicate with each other before the start of the experiment. All earnings and the information cost in the game were measured in terms of points and an exchange rate of 100 centavos=40 points was used.[6]

The instructions of the game were presented on the computer in Spanish and were also read aloud by the research assistant. Then the game was started. After each match players were told what strategies the other player used (including whether he or she bought information) and players were rematched. After 5 rounds players were debriefed. Basic demographic data was collected from 346 subjects: 58 percent of subjects were male and the average student was 22.9 years old. Subjects were paid in cash at the end of the experiment.

3.3 Results

The results are summarized in table 4 for all three treatments. I compare strategies of players both between the three treatments and between early rounds (1-3) and late rounds (4-5). Starred entries indicate that for early (late) rounds the corresponding entry in the C25/C5 treatment is significantly different from the entry in treatment NI. Spade entries indicate significant difference between early and late round play (all at 5 percent level).

The figure shows several trends. First of all, in the no information treatment NI average play converges to the mixed Nash equilibrium in later rounds where each player plays her

[6]The earnings from the experiment were significant as the average hourly wage rate for students was around 7 pesos.

Figure 4: Comparison of early and late round play in sequential Battle of the Sexes game with information costs. Starred entries indicate significant differences between C25/C5 and NI treatments while spade entries indicate significant differences between early and late play.

Rounds 1 to 3

Treatment C25 – Infocost 25

	$0.48^{\spadesuit}$ **A**	$0.52^{\spadesuit}$ **B**
$0.78^{\spadesuit}$ **A**	0.55	0.23
$0.22^{\spadesuit}$ **B**	0.09	0.13

61% of followers buy information

Treatment C5 – Infocost 5

	$0.46^{\spadesuit}$ **A**	$0.54^{\spadesuit}$ **B**
0.73 **A**	0.57	0.16
0.27 **B**	0.06	0.20

67% of followers buy information

Treatment NI – No Info

	0.47 **A**	0.53 **B**
$0.70^{\spadesuit}$ **A**	0.30	0.40
$0.30^{\spadesuit}$ **B**	0.17	0.13

Rounds 4 to 5

Treatment C25 – Infocost 25

	$0.63^{\spadesuit,*}$ **A**	$0.37^{\spadesuit,*}$ **B**
$0.88^{\spadesuit,*}$ **A**	0.68	0.19
$0.13^{\spadesuit,*}$ **B**	0.03	0.09

57% of followers buy information

Treatment C5 – Infocost 5

	$0.56^{\spadesuit,*}$ **A**	$0.44^{\spadesuit,*}$ **B**
0.72^{*} **A**	0.57	0.15
0.28^{*} **B**	0.06	0.21

71% of followers buy information

Treatment NI – No Info

	0.43^{*} **A**	0.57^{*} **B**
$0.58^{\spadesuit,*}$ **A**	0.28	0.30
$0.42^{\spadesuit,*}$ **B**	0.16	0.27

preferred strategy with approximate probability $\frac{2}{3}$. While there appears to be some framing due to the move order on the leader's preferred action initially this effect dissipates over time. The degree of coordination on (A,A) and (B,B) is close to the prediction of $\frac{4}{9}$. This observation is important because we can interpret deviations from the mixed Nash outcome in the C5 and C25 treatments as coming from the availability of information rather than the framing of the move order.

In the C5 and C25 treatments players converge to the Stackelberg outcome over time and compared to treatment NI. Both the leader and the follower tend to play the leader's preferred strategy more often in later rounds. Coordination therefore rises to almost 80 percent probability and hence almost doubles. This is consistent with the irrationality hypothesis but not with the commitment hypothesis.

One might expect that the irrationality hypothesis should be stronger in the C5 treatment compared to the C25 treatment because information is less expensive. I find mixed support for this hypothesis. Significantly more subjects purchase information in the C5 treatment both in early and late rounds which is consistent with the irrationality hypothesis. On the other hand, while the percentage of games which result in miscoordination is similar in both the C5 and the C25 treatments (about 20 percent) there is slightly more coordination on the (B,B) outcome in treatment C5 which is inconsistent with the irrationality hypothesis. However, coordination on (A,A) is still more than double as high in the C5 treatment compared to the NI treatment.

the trend over time suggest coordination on the Stackelberg outcome (A,A) which is consistent with the irrationality hypothesis.

4 Conclusion

I show how small information costs weaken the clean predictions of subgame perfect equilibrium. I provide a simple algorithm to find the strategy profiles which we expect to possibly see being played when information costs are small but positive. Even though player 1 essentially plays a pure strategy in such an environment, she does not necessarily play her Stackelberg strategy any longer.

However, the experimental evidence also suggests that in practice the Stackelberg equilibrium might be more robust than predicted by my theory as long as there are sufficiently many irrational players in the population.

References

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A Proofs

A.1 Proof of Proposition 1

This follows immediately from the fact that Γ_ϵ is non-empty. Hence there exist at least one sequence (σ_{ϵ_n}) which has a convergence subsequence because it is bounded in a finite-dimensional space.

A.2 Proof of Proposition 2

Trembling-hand perfection and the assumption that the base game is generic ensures that player 2 will always buy information since it is costless. Therefore the only possible equilibrium has player 2 play a best-response to each of player 1's actions. Therefore player 1 will play his Stackelberg strategy.

A.3 Proof of Proposition 3

Since information is costly player 2 will not buy any information if player 1 plays a pure strategy. This implies the actions s_1 of player 1 and s_2 of player 2 along the equilibrium path are a Nash equilibrium of the base game. For any $\sigma \in \tilde{\Gamma}_P \subset \Gamma_\epsilon$ the sequence (σ) converges to $\sigma \in \Gamma^*$.

A.4 Proof of Proposition 4

We know that player 1 mixes over more than one strategy in a mixed equilibrium. Assume player 2 buys no information. In a mixed equilibrium player 2 has to play at least two strategies - otherwise player 1 has no incentive to play a mixed strategy by assumption (3).

Player 2 is indifferent between all strategies in her support $supp(\sigma_2)$. Each action $b_k \in supp(\sigma_2)$ is with probability γ_k a best response to any strategy over which player 1 mixes according to his mixed strategy σ_1. Note, that $\gamma_k > 0$ - otherwise b_k would not be in player 2's support. We know that $\sum_{b_k \in supp(\sigma_2)} \gamma_k = 1$. Hence there is at least one strategy in player 2's support which at least 50 percent of the time is *not* a best response to a strategy chosen by player 1. Hence if $\epsilon \leq \frac{m}{2}$ the follower does strictly better if he buys information.

A.5 Proof of Lemma 1

Assume player 2 would buy information for sure. Then player 1 would play his Stackelberg strategy for sure. By proposition 3 player 2 should not buy information which is a contradiction.

A.6 Proof of Theorem 1

Assume player 1 mixes over strategies a_{i_k} $(k = 1, 2, .., K)$ with strictly positive probability α_{i_k} and that b is in the support of player 1's equilibrium strategy. Player 2 is indifferent between buying information and not buying information:

$$\sum_{k=1}^{K} \alpha_{i_k} v\left(a_{i_k}, b\right) = \sum_{k=1}^{K} \alpha_{i_k} v\left(a_{i_k}, BR_2(a_{i_k}\right) - \epsilon \tag{2}$$

There is at least one strategy in the support of player 1 where player 2 would prefer to deviate from b. The total probability weight on these actions is denoted with α^*. We then have

$$\alpha^* m < \epsilon \tag{3}$$

which implies $\alpha^* < \frac{\epsilon}{m}$.

Therefore any strategy in the follower's support (if she buys no information) is a best response to the leader's strategy with probability of at least $1 - n_1 \frac{\epsilon}{m}$. But since the best response of the follower is unique it follows that for any mixed strategy of the leader the follower will strictly prefer buying information to not buying information and play a strictly mixed strategy if $\epsilon < (1 - n_1 \frac{\epsilon}{m}) \frac{m}{2}$. Therefore, there is some ϵ^* such that for $\epsilon < \epsilon^*$ the follower plays a pure strategy b if she does not buy information.

Hence player 1 mixes with probability of at least $1 - n_1 \frac{\epsilon}{m}$ over a set of strategies to which b is a best response. The leader will prefer the strategy which provides her with the greater utility for ϵ sufficiently small.

To make player 1 indifferent between his main strategy a and some other strategy a' to which b is not a best response the following has to hold:

$$u(a, b) = (1 - \beta) u(a', b) + \beta u(a', BR_2(a')) + O(\epsilon) \tag{4}$$

For each a we can find such an equation and the associated β_a with which player 2 buys information. We take the minimum and maximum over all these β which will give us upper and lower bounds $\underline{\beta}$ and $\overline{\beta}$.

A.7 Proof of Lemma 2

A subsequence of pure strategies will always converge to a pure strategy while a subsequence of mixed strategies can never converge to an element of $\tilde{\Gamma}_P$ because $\underline{\beta} < \beta < \overline{\beta}$. Finally, theorem 1 guarantees that any sequence of mixed equilibria where the cost of information $\epsilon_n \to 0$ can only have limit points where both players 1 and 2 play pure strategies.

A.8 Proof of Theorem 2

We compare the payoffs of player 1 if he sticks to his strategy a and if he switches to some alternative $a' \in A_L(a)$. He will prefer to keep playing a if

$$\begin{aligned} a &> \beta u(a', b) + (1-\beta) u(a', BR_2(a')) \\ \beta &> \frac{u(a,b) - u(a', BR_2(a')}{u(a', b) - u(a', BR_2(a')} \end{aligned} \tag{5}$$

This gives us the condition $\beta > \beta_L$. Similarly, we find that player 1 will not deviate to some action $a' \in A_H(a)$ if and only if $\beta < \beta_H$.

A.9 Proof of Proposition 5

If the Stackelberg equilibrium is a pure Nash equilibrium the claim follows from proposition 3. Otherwise, player 1 would like to deviate to some other strategy a' which gives him higher payoff. However, player 2's best response to a' cannot be the same as to a - otherwise player 1 would always play a'. We also have $u(a', BR_2(a')) < u(a, b)$ because a is the Stackelberg strategy of player 1. This implies that $\beta_H > 0$. We can also show that $A_L(a)$ is empty and hence $\beta_L = 0$: whenever player 1 deviates to a strategy a' such that $u(a, b) < u(a, b)$ and player 2 plays his best response then $u(a', BR_2 0(a')) < u(a, b)$ because a is the Stackelberg strategy. Theorem 2 now allows us to show that the Stackelberg equilibrium σ_S is in Γ^*.

A Spanish Instructions - Infocost 25

Player 1 - infocost=25 points - instruction screen

Instrucciones del Juego: Jugador 1

Está por participar en un experimento de toma de decisiones. Este experimento es parte de un proyecto de inversión financiado por las Unversidades de Harvard y Wesleyan. Por presentarse en tiempo y hora le será pagado una tasa de participación de 12 Pesos. Esta tasa es la misma para todos los participantes. Además puede incrementar esta suma durante el experimento. Este dinero (tasa de participación + ganacias adicionales) le será pagado al final del experimento y en forma privada.

Por favor lea las instrucciones restantes cuidadosamente. Toda información que reciba de nosotros es para uso privado, sin excepciones. No está permitido que pase ningún tipo de información a otros participantes del experimento. No está permitido hablar durante el experimento. La violación de cualquiera de estas reglas nos obligará a parar el experimento. Si tiene alguna pregunta, por favor hágala antes de comenzar el experimento.

Este juego consiste de 5 rondas. En cada ronda competirá contra un Jugador de esta sala, seleccionado en forma aleatoria. Usted será un Jugador 1 durante este juego y jugará contra un Jugador 2, elegido de este grupo en forma aleatoria. Ambos tendrán que elegir entre dos posibles acciones: A y B. Usted elegirá primero. El jugador 2 no podrá observar esa elección en forma directa. El jugador 2 tendrá la posibilidad de observar su elección, a un coste de de 25 Puntos antes de decidir. Si el Jugador 2 elige no observar su acción él pagará 0 Puntos. Luego elegirá su acción (A o B).

Si ambos jugadores eligen A, usted obtendrá 500 Puntos y el jugador 2 ganará 250 Puntos. Si ambos eligen B, usted ganará 250 Puntos y el jugador 2 obtendrá 500 Puntos. Si ambos eligen diferentes acciones obtendrán 0 (cero) Puntos cada uno.

Estos son los pagos de acuerdo con lo que elija (A o B) cada Jugador:

Recuerde que usted es un **Jugador 1**.

		Jugador 2: A	Jugador 2: B
Jugador 1	A	**500** , 250	**0** , 0
	B	**0** , 0	**250** , 500

Si comprendió estas instrucciones por favor presione "Seguir" para comenzar el Juego.

Seguir

Player 2 - infocost=25 points - instruction screen

Instrucciones del Juego: Jugador 2

Está por participar en un experimento de toma de decisiones. Este experimento es parte de un proyecto de inversión financiado por las Unversidades de Harvard y Wesleyan. Por presentarse en tiempo y hora le será pagado una tasa de participación de 12 Pesos. Esta tasa es la misma para todos los participantes. Además puede incrementar esta suma durante el experimento. Este dinero (tasa de participación + ganacias adicionales) le será pagado al final del experimento y en forma privada.

Por favor lea las instrucciones restantes cuidadosamente. Toda información que reciba de nosotros es para uso privado, sin excepciones. No está permitido que pase ningún tipo de información a otros participantes del experimento. No está permitido hablar durante el experimento. La violación de cualquiera de estas reglas nos obligará a parar el experimento. Si tiene alguna pregunta, por favor hágala antes de comenzar el experimento.

Este juego consiste de 5 rondas. En cada ronda competirá contra un Jugador de esta sala, seleccionado en forma aleatoria. Usted será el Jugador 2 durante este juego y jugará contra un Jugador 1, elegido de este grupo en forma aleatoria. Ambos tendrán que elegir entre dos posibles acciones: A y B. El Jugador 1 elegirá primero. Usted no podrá observar esa elección en forma directa. Usted tendrá la posibilidad de observar la elección del Jugador 1, a un coste de de 25 Puntos, antes de decidir. Si usted elige no observar la acción del Jugador 1 usted pagará 0 Puntos. Luego usted elegirá su acción (A o B).

Si ambos jugadores eligen A, usted obtendrá 250 Puntos y el Jugador 1 ganará 500 Puntos. Si ambos eligen B, usted ganará 500 Puntos y el Jugador 1 obtendrá 250 Puntos. Si ambos eligen diferentes acciones obtendrán 0 (cero) Puntos cada uno.

Estos son los pagos de acuerdo con lo que elija (A o B) cada Jugador:

Recuerde que usted es un **Jugador 2**.

		Jugador 2: A	Jugador 2: B
Jugador 1	A	**500**, **250**	**0** , **0**
	B	**0** , **0**	**250** ,**500**

Si comprendió estas instrucciones por favor presione "Seguir" para comenzar el Juego.

Seguir

Fase del Juego: Jugador 1

Usted es el Jugador 1 y tiene que elegir una acción entre A o B.

- ○ **No seleccionó una acción**
- ◉ **Acción A**
- ○ **Acción B**

Seguir

Player 1 - infocost=25 points - decision screen

Esperando a que el jugador 1 elija.

Player 2 - information cost=25 Points - waiting screen

Player 1 - infocost=25 points - waiting screen

Esperando a que el jugador 2 elija.

Player 2 - infocost=25 points - information screen

Fase del Juego: Jugador 2

El Jugador 1 ya eligió. Quiere conocer esta decisión a un costo de 25 Puntos?

○ **No seleccionó una acción**
◉ **Si**
○ **No**

Seguir

Player 1 - infocost=25 points - waiting screen

Esperando a que el jugador 2 elija.

Player 2 - information cost=25 Points - decision screen

Fase del Juego: Jugador 2

Usted es el Jugador 2 y decidió enterarse de la elección del Jugador 1. El Jugador 1 eligió la acción A. Por favor elija entra la acción A o B.

○ **No seleccionó una acción**
◉ **Acción A**
○ **Acción B**

Seguir

Player 1 - infocost=25 points - feedback screen

Resultados de la Ronda 1

Usted es el Jugador 1 y ha elegido la acción A. El Jugador 2 compró información y eligió la acción A. En esta ronda obtuvo 500 Puntos y el Jugador 2 obtuvo 225 Puntos.

Si entendió los resultados de esta ronda, por favor apriete el boton inferior.

Ir a la Ronda Siguiente

Player 2 - information cost=25 Points - feedback screen

Resultados de la Ronda 1

Usted es el Jugador 2. El Jugador 1 eligió la acción A. Usted compró información y eligió la acción A. En esta ronda el Jugador 1 obtuvo 500 Puntos y usted obtuvo 225 Puntos.

Si entendió los resultados de esta ronda, por favor apriete el boton inferior.

Ir a la Ronda Siguiente

B English Translations of Spanish Instructions - Infocost 25

Player 1 - infocost=25 points - instruction screen

Instructions of the Game: Player 1

You are about to to participate in an experiment on decision making. This experiment is part of a research project financed by Harvard and Wesleyan University. You will receive a participation reward of 12 Pesos for appearing on time. All participants will receive this. In addition you can increase your earnings during the experiment. You total earnings (participation reward plus earnings generated during the experiment) will be paid to you privately at the end of the experiment.

Please read these instructions carefully. Please make decisions privately and do not talk to other participants in this room. If you do not follow these rules we will have to stop the experiment. If you have any questions please ask us before the experiment starts.

This game consists of 5 rounds. In each round you will play with one randomly selected player in this room. You will play the role of Player 1 in this game and you will play with players who play the role of player 2. Both of you will have to choose between two possible actions: A and B. You will choose first. Player 2 cannot observe your action directly. However, player 2 has the opportunity to observe your action at a cost of 25 Points, before deciding on his action. If he chooses not to observe your action he will pay 0 points. Then player 2 will choose his action (A or B).

If both players choose action A, you will obtain 500 Points and player 2 will earn 250 Points. If both choose B, you will gain 250 Points and player 2 will obtain 500 Points. If both of you choose different actions you both will obtain 0 (zero) Points.

The following diagram shows again the payments which each player receives when playing action A or B:

Remember that you are **Player 1**.

		Player 2	
		A	B
Player 1	A	**500** , 250	**0** , 0
	B	**0** , 0	**250** , 500

If you understood these instructions please press "Continue" to begin the Game.

Continue

Player 2 - infocost=25 points - instruction screen

Instructions of the Game: Player 2

You are about to to participate in an experiment on decision making. This experiment is part of a research project financed by Harvard and Wesleyan University. You will receive a participation reward of 12 Pesos for appearing on time. All participants will receive this. In addition you can increase your earnings during the experiment. You total earnings (participation reward plus earnings generated during the experiment) will be paid to you privately at the end of the experiment.

Please read these instructions carefully. Please make decisions privately and do not talk to other participants in this room. If you do not follow these rules we will have to stop the experiment. If you have any questions please ask us before the experiment starts.

This game consists of 5 rounds. In each round you will play with one randomly selected player in this room. You will play the role of Player 2 in this game and you will play with players who play the role of player 1. Both of you will have to choose between two possible actions: A and B. Player 1 will choose first. You cannot observe his action directly. However, you have the opportunity to observe player 1's action at a cost of 25 Points, before deciding on your action. If you choose not to observe player 1's action you will pay 0 points. Then you will choose your action (A or B).

If both players choose action A, you will obtain 250 Points and player 1 will earn 500 Points. If both choose B, you will gain 500 Points and player 1 will obtain 250 Points. If both of you choose different actions you both will obtain 0 (zero) Points.

The following diagram shows again the payments which each player receives when playing action A or B:

Remember that you are **Player 2**.

		Player 2	
		A	B
Player 1	A	**500**, **250**	**0** , **0**
	B	**0** , **0**	**250** ,**500**

If you understood these instructions please press "Continue" to begin the Game.

Continue

Player 1 - infocost=25 points - decision screen

Game running: Player 1

You are Player 1 and must choose action A or B.

- ( ) No action selected
- (•) Action A
- ( ) Action B

[Seguir]

Player 2 - infocost=25 points - waiting screen

Waiting for player 1 to make his choice.

Waiting for player 2 to make his choice.

Player 1 - infocost=25 points - waiting screen

Game running: Player 2

Player 1 has already chosen his action. Do you want to know his choice at a cost of 25 Points?

- ○ No selection
- ◉ Yes
- ○ No

Continue

Player 2 - infocost=25 points - information screen

Player 1 - infocost=25 points - waiting screen

Waiting for player 2 to make his choice.

Player 2 - information cost=25 Points - decision screen

Game running: Player 2

You are Player 2 and decided to find out the choice of Player 1. Player 1 chose the action A. Please choose action A or B.

- ○ **No action selected**
- ◉ **Action A**
- ○ **Action B**

Seguir

Results of Round 1

You are player 1 and have selected action A. Player 2 has bought information about your action and chose action A. In this round you have earned 500 Points and Player 2 has earned 225 Points.

If you understand the results in this round please click the button below.

Go to next round

Player 1 - infocost=25 points - feedback screen

Results of Round 1

You are player 2. Player 1 has chosen action A. You have bought information and have chosen action A. In this round player 1 has earned 500 Points and you have earned 225 Points.

If you understand the results in this round please click the button below.

Go to next round

Player 2 - information cost=25 Points - feedback screen

C Spanish Instructions - No Information treatment

Player 1 - no information - instruction screen

Instrucciones del Juego: Jugador 1

Está por participar en un experimento de toma de decisiones. Este experimento es parte de un proyecto de inversión financiado por las Unversidades de Harvard y Wesleyan. Por presentarse en tiempo y hora le será pagado una tasa de participación de 12 Pesos. Esta tasa es la misma para todos los participantes. Además puede incrementar esta suma durante el experimento. Este dinero (tasa de participación + ganacias adicionales) le será pagado al final del experimento y en forma privada.

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Este juego consiste de 5 rondas. En cada ronda competirá contra un Jugador de esta sala, seleccionado en forma aleatoria. Usted será un Jugador 1 durante este juego y jugará contra un Jugador 2, elegido de este grupo en forma aleatoria. Ambos tendrán que elegir entre dos posibles acciones: A y B. Usted elegirá primero. El jugador 2 no podrá observar esa elección. Luego elegirá su acción (A o B).

Si ambos jugadores eligen A, usted obtendrá 500 Puntos y el jugador 2 ganará 250 Puntos. Si ambos eligen B, usted ganará 250 Puntos y el jugador 2 obtendrá 500 Puntos. Si ambos eligen diferentes acciones obtendrán 0 (cero) Puntos cada uno.

Estos son los pagos de acuerdo con lo que elija (A o B) cada Jugador:

Recuerde que usted es un **Jugador 1**.

		Jugador 2: A	Jugador 2: B
Jugador 1	A	**500** , 250	**0** , 0
	B	**0** , 0	**250** , 500

Si comprendió estas instrucciones por favor presione "Seguir" para comenzar el Juego.

Seguir

Player 2 - no information - instruction screen

Instrucciones del Juego: Jugador 2

Está por participar en un experimento de toma de decisiones. Este experimento es parte de un proyecto de inversión financiado por las Unversidades de Harvard y Wesleyan. Por presentarse en tiempo y hora le será pagado una tasa de participación de 12 Pesos. Esta tasa es la misma para todos los participantes. Además puede incrementar esta suma durante el experimento. Este dinero (tasa de participación + ganacias adicionales) le será pagado al final del experimento y en forma privada.

Por favor lea las instrucciones restantes cuidadosamente. Toda información que reciba de nosotros es para uso privado, sin excepciones. No está permitido que pase ningún tipo de información a otros participantes del experimento. No está permitido hablar durante el experimento. La violación de cualquiera de estas reglas nos obligará a parar el experimento. Si tiene alguna pregunta, por favor hágala antes de comenzar el experimento.

Este juego consiste de 5 rondas. En cada ronda competirá contra un Jugador de esta sala, seleccionado en forma aleatoria. Usted será el Jugador 2 durante este juego y jugará contra un Jugador 1, elegido de este grupo en forma aleatoria. Ambos tendrán que elegir entre dos posibles acciones: A y B. El Jugador 1 elegirá primero. Usted no podrá observar esa elección. Luego usted elegirá su acción (A o B).

Si ambos jugadores eligen A, usted obtendrá 250 Puntos y el Jugador 1 ganará 500 Puntos. Si ambos eligen B, usted ganará 500 Puntos y el Jugador 1 obtendrá 250 Puntos. Si ambos eligen diferentes acciones obtendrán 0 (cero) Puntos cada uno.

Estos son los pagos de acuerdo con lo que elija (A o B) cada Jugador:

Recuerde que usted es un **Jugador 2**.

		Jugador 2: A	Jugador 2: B
Jugador 1	A	**500**, **250**	**0** , **0**
	B	**0** , **0**	**250** ,**500**

Si comprendió estas instrucciones por favor presione "Seguir" para comenzar el Juego.

Seguir

Fase del Juego: Jugador 1

Usted es el Jugador 1 y tiene que elegir una acción entre A o B.

- ( ) **No seleccionó una acción**
- (•) **Acción A**
- ( ) **Acción B**

Seguir

Player 1 - no information - decision screen

Esperando a que el jugador 1 elija.

Player 2 - no information - waiting screen

Player 1 - no information - waiting screen

Esperando a que el jugador 2 elija.

Player 2 - no information - decision screen

Fase del Juego: Jugador 2

Usted es el Jugador 2, por favor elija entre la acción A o B.

- ○ **No seleccionó una acción**
- ◉ **Acción A**
- ○ **Acción B**

Seguir

Resultados de la Ronda 1

Usted es el Jugador 1 y ha elegido la acción A. El Jugador 2 eligió la acción A. En esta ronda obtuvo 500 Puntos y el Jugador 2 obtuvoa 250 Puntos.

Si entendió los resultados de esta ronda, por favor apriete el boton inferior.

Ir a la Ronda Siguiente

Player 1 - no information - feedback screen

Resultados de la Ronda 1

Usted es el Jugador 2. El Jugador 1 eligió la acción A. Usted eligió la acción A. En esta ronda el Jugador 1 obtuvo 500 Puntos y usted obtuvo 250 Puntos.

Si entendió los resultados de esta ronda, por favor apriete el boton inferior.

Ir a la Ronda Siguiente

Player 2 - no information - feedback screen

D English Translations of Spanish Instructions - No Information Treatment

Player 1 - no information - instruction screen

Instructions of the Game: Player 1

You are about to to participate in an experiment on decision making. This experiment is part of a research project financed by Harvard and Wesleyan University. You will receive a participation reward of 12 Pesos for appearing on time. All participants will receive this. In addition you can increase your earnings during the experiment. You total earnings (participation reward plus earnings generated during the experiment) will be paid to you privately at the end of the experiment.

Please read these instructions carefully. Please make decisions privately and do not talk to other participants in this room. If you do not follow these rules we will have to stop the experiment. If you have any questions please ask us before the experiment starts.

This game consists of 5 rounds. In each round you will play with one randomly selected player in this room. You will play the role of Player 1 in this game and you will play with players who play the role of player 2. Both of you will have to choose between two possible actions: A and B. You will choose first. Player 2 cannot observe your action. Then player 2 will choose his action (A or B).

If both players choose action A, you will obtain 500 Points and player 2 will earn 250 Points. If both choose B, you will gain 250 Points and player 2 will obtain 500 Points. If both of you choose different actions you both will obtain 0 (zero) Points.

The following diagram shows again the payments which each player receives when playing action A or B:

Remember that you are **Player 1**.

		Player 2: A	Player 2: B
Player 1	A	**500** , 250	**0** , 0
	B	**0** , 0	**250** , 500

If you understood these instructions please press "Continue" to begin the Game.

Continue

Player 2 - no information - instruction screen

Instructions of the Game: Player 2

You are about to to participate in an experiment on decision making. This experiment is part of a research project financed by Harvard and Wesleyan University. You will receive a participation reward of 12 Pesos for appearing on time. All participants will receive this. In addition you can increase your earnings during the experiment. You total earnings (participation reward plus earnings generated during the experiment) will be paid to you privately at the end of the experiment.

Please read these instructions carefully. Please make decisions privately and do not talk to other participants in this room. If you do not follow these rules we will have to stop the experiment. If you have any questions please ask us before the experiment starts.

This game consists of 5 rounds. In each round you will play with one randomly selected player in this room. You will play the role of Player 2 in this game and you will play with players who play the role of player 1. Both of you will have to choose between two possible actions: A and B. Player 1 will choose first. You cannot observe his action. Then you will choose your action (A or B).

If both players choose action A, you will obtain 250 Points and player 1 will earn 500 Points. If both choose B, you will gain 500 Points and player 1 will obtain 250 Points. If both of you choose different actions you both will obtain 0 (zero) Points.

The following diagram shows again the payments which each player receives when playing action A or B:

Remember that you are **Player 2**.

		Player 2: A	**Player 2**: B
Player 1	A	**500, 250**	**0 , 0**
	B	**0 , 0**	**250 ,500**

If you understood these instructions please press "Continue" to begin the Game.

Continue

Game running: Player 1

You are Player 1 and must choose action A or B.

No action selected
Action A
Action B

Seguir

Player 1 - no information - decision screen

Waiting for player 1 to make his choice.

Player 2 - no information - waiting screen

Player 1 - no information - waiting screen

Waiting for player 2 to make his choice.

Player 2 - no information - decision screen

Game running: Player 2

You are Player 2. Please choose action A or B.

- ○ **No seleccionó una acción**
- ● **Acción A**
- ○ **Acción B**

Seguir

Results of Round 1

You are player 1 and have selected action A. Player 2 has chosen action A. In this round you have earned 500 Points and Player 2 has earned 250 Points.

If you understand the results in this round please click the button below.

Go to next round

Player 1 - no information - feedback screen

Results of Round 1

You are player 2. Player 1 has chosen action A. You have chosen action A. In this round player 1 has earned 500 Points and you have earned 250 Points.

If you understand the results in this round please click the button below.

Go to next round

Player 2 - no information - feedback screen