On the Transition Between Monetary Regimes*

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Abstract

Historically, commodity money preceded fiat money. Standard search-theoretical models of money such as Kiyotaki and Wright (1989) cannot explain this transition because of multiple equilibria: a small infusion of fiat money with superior intrinsic characteristics into a commodity money equilibrium is always valued if agents believe in its acceptability. We propose a natural extension of the standard model in order to break this indeterminacy. We assume (1) that agents derive positive utility from consuming even non-favorite commodities and (2) that agents have to consume regularly. We find that agents accept only commodity money if search frictions are large. Fiat money can become valuable in sufficiently advanced economies with small search frictions.

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1 Introduction

Search theory has become a useful tool to explore the medium of exchange role of money. This line of research started with the seminal paper by Kiyotaki and Wright (1989) who examined how the intrinsic properties of goods make them more or less likely to emerge as commodity monies. Such properties include various forms of storability, durability, or portability (Kiyotaki and Wright 1989, Aiyagari and Wallace 1991), homogeneity or recognizability (Williamson and Wright 1994, Cuadras-Morato 1994, Li 1995), and the relative supply and demand of goods (Cuadras-Morato and Wright 1997). Kiyotaki and Wright (1989) also show that fiat money with intrinsic properties at least as good as the best available commodity money can be valued, and they generalize this result in a more tractable framework in Kiyotaki and Wright (1993).

In some ways these models are too successful in explaining the existence of fiat money. In the standard Kiyotaki-Wright (1989) model a small infusion of fiat money with superior intrinsic characteristics into a commodity money equilibrium is always valued if agents believe in its acceptability. The introduction of fiat money at the margin does not change the incentives of agents to exchange commodities, but agents strictly prefer to exchange a commodity good for money because of its superior intrinsic properties (see Aiyagari and Wallace (1991) for a generalization of this observation to an N good economy).

Nevertheless, fiat money arose historically only in advanced economies such as ancient China (see Tullock (1957)), and truly inconvertible fiat money became prevalent in most of the world only as late as the 20th century. Why did it take so many years for pure fiat money to emerge even though the basic concept is simple to implement? Ritter (1995) analyzes the incentives of government to produce fiat money and argues, that the government has to be sufficiently big to introduce fiat money in a credible manner. Internalizing some of the macroeconomic benefits of fiat money can offset the benefits of seignorage such that the government will not debase the currency through excessive money printing. We are therefore more likely to observe fiat money in more advanced economies because the government has assumed enough functions to internalize the benefits of fiat money.

In this paper we propose an alternative explanation for the emergence of fiat money in more advanced economies by equating greater economic development with s smaller degree of search frictions in the exchange process. The Poisson matching process in the standard Kiyotaki-Wright model (and its derivatives) suppresses the role of search frictions. Whenever the matching rate is sufficiently high to support a barter equilibrium there is also a fiat money equilibrium. We break this link by making two natural assumptions. First, agents derive some positive base utility from consuming any commodity but enjoy extra utility when they consume their favorite consumption good. This set-up preserves the incentive to engage in trade and the "double coincidence of wants" problem. In addition, it creates a potential insurance motive for holding commodity versus fiat money because commodity money can be consumed at any time while the holder of fiat money can only consume after she has exchanged it for a commodity. However, this assumption on its own does not resolve the indeterminacy of the model because in equilibrium agents will never want to consume any good other than their favorite consumption good. At any point in time their decision to engage in exchange reveals their preference for waiting rather than for consuming immediately. This is a consequence of the Poisson structure of the Kiyotaki-Wright model: the optimal strategies of agents are not influenced by the length of time they have spent in the market. We move beyond the Poisson set-up by requiring that agents have to consume in *regular intervals*, a standard assumption in inventory-theoretic models of money demand such as Baumol (1952) and Tobin (1956). In such a world agents who have just consumed may be willing to accept fiat money because of its superior intrinsic properties. However, agents who have not consumed for a while will be more reluctant to do the same and will prefer the better insurance provided by commodities to a risk of not finding a suitable trading partner. The unwillingness of some 'late' agents to accept fiat money in return reduces the overall marketability of fiat money and the incentives of 'early' agents to accept it. In some cases this feedback can unravel any fiat money equilibrium - agents will therefore never accept the infusion of such money into the economy.

The crucial parameter in our model is the degree of search frictions, i.e. the rate at which agents are matched. In an economy with large search frictions agents will only accept commodity money. Only a more developed economy with smaller frictions can value an infusion of fiat money.

A simple example might be useful to clarify the story we tell in our model. Assume a large number of hikers enter Yellowstone National Park each day at random times for a 24 hour hike. Exactly $n-1$ of them carry one piece of fruit which is not their favorite one, and one hiker has a Susan B. Anthony Dollar. If a hiker does not eat at all until the end of the hike she will drive home hungry and frustrated and enjoy zero utility from the hike. If she eats her favorite fruit she enjoys a utility of 1, and every other fruit yields a payoff $s < 1$. At any point in time our Yellowstone fruit economy is therefore populated by a large number of hungry hikers who are eager to trade with each other. It is easy to see that each fruit can serve as commodity money in this symmetric set-up: a hiker does not suffer a loss from exchanging one piece of non-favorite fruit for another until she receives her favorite one. The interesting question is whether the holders of Susan B. Anthony Dollars can ever eat. If the number of hikers is small, every hiker will be reluctant to accept the Dollar because he is afraid not to find a subsequent trading partner until the end of the hike. However, if there are many hikers, they will run into each other more frequently and might be willing to accept the Susan B. Anthony Dollar in exchange for their endowment at least early in their hike (the coin is much lighter to carry). Even though some hikers will still get stuck with the non-digestible Dollar, this risk will only become a concern shortly before the end of their hike.

The remainder of the paper is organized as follows. In section 2 we introduce the formal model and show that all commodities serve as commodity monies in indirect exchange. The main result on the existence of fiat money equilibria is derived in section 3. Section 4 concludes.

2 The Economy

The basic set-up of the model follows Kiyotaki-Wright (1989). Time is continuous and infinite, and at each point in time there are three indivisible commodities which we call goods 1, 2 and 3. A unit mass of infinitely lived agents specialize both in consumption and production, with equal proportions of type 1, 2 and 3 agents. Type i agents are only able to produce some good $i^* \neq i$, and good i is their favorite consumption good. There is also some indivisible good 0 which we refer to as fiat money and which can be costlessly disposed of at any point in time.

Fiat money is in zero supply: we are interested whether agents would accept the injection of fiat money into the economy at the margin. Agents can only carry one unit of any good at any point in time.

Agents can either stay at home, or participate in exchange in order to trade their inventory for their favorite consumption good. Agents who carry one of the three commodities to the market incur a flow cost of c, while money is costless to carry. Exchange is accomplished through random pairwise matching at rate a. Trade is always bilateral and must entail a one-for-one swap of inventories. Note that because of the "double coincidence of wants" problem some agents have to engage either in indirect exchange or accept fiat money. We assume that the swap of inventories does not incur any transaction costs. Agents will therefore agree to an exchange if they are indifferent as to which of the two goods they hold.

Our model differs significantly from the Kiyotaki-Wright framework in the way agents consume their endowments. We assume that agents have to consume regularly at times t, $t+1$ and so on. We denote the time which has passed since the last consumption opportunity with $\tilde{t} \in [0, 1]$. Whenever an agent of type i consumes, she can immediately produce a new unit of good i^* at no cost. We assume that the consumption opportunities of agents are uniformly distributed such that consumption occurs at a steady rate. This assumption allows us to restrict attention to steady state equilibria of the model.

Agents derive one unit of utility from consuming their favorite good and s units of utility from consuming some other commodity ($0 < s < 1$). Fiat money is intrinsically worthless and agents receive 0 units of utility from eating it. Agents do not discount between consumption opportunities and they maximize the utility they obtain from their next consumption opportunity. Agent's i utility can be expressed as

$$E\left[I_F + sI_{NF} - \int_0^1 cI_G\left(\tilde{t}\right)d\tilde{t}\right], \tag{1}$$

where I_F and I_{NF} are random indicator functions that equal one if the agent consumes her favorite commodity/ non-favorite commodity, and zero otherwise. The indicator function $I_G\left(\tilde{t}\right)$ equals 1 if the agent carries a commodity at time $\tilde{t}$ and participates in exchange.[1]

As in Kiyotaki and Wright (1989), our environment is time-invariant and we restrict attention to steady-state, pure-strategy equilibria. The strategies of each agent i are defined by the the participation rule $p_i\left(j,\tilde{t}\right)$ and the trading rule $\tau_i\left(j,k,\tilde{t}\right)$. If the agent holds good j after time $\tilde{t}$ and agrees to participate in the market we set $p\left(j,\tilde{t}\right)=1$ and zero otherwise. Let $\tau_i\left(j,k,\tilde{t}\right)=1$ if the agent agrees to swap good j for good k at time $\tilde{t}$, and zero otherwise. Because our model is completely symmetric we concentrate on symmetric trading equilibria such that $\tau_i\left(j,k,\tilde{t}\right)=1$ for $j,k\notin\{0,i\}$ (e.g., agents willingly swap two commodity goods which are not their favorite consumption goods).

For an equilibrium to exist, it is important that there are gains from trade: agents who hold a commodity have to prefer participation in the exchange process to staying out of the market. The probability that a participating agent is matched with someone during a small time interval $\Delta\tilde{t}$ is $a\Delta\tilde{t}$. That agent will carry her favorite consumption good with probability $\frac{1}{3}$ due to the symmetry of

[1] This implies that agents completely discount future consumption. We also looked at an alternative specification where agents discount future consumption by a discount factor $0<\beta<1$ and their life-time utility is

$$E\sum_{t=0}^{\infty}\beta^t\left[I_F(t) + sI_{NF}(t) - \int_0^1 cI_G\left(t,\tilde{t}\right)d\tilde{t}\right].$$

The analysis becomes more complicated but the basic results are unaffected.

the equilibrium. Therefore, the gross gain from participation is $\frac{a}{3}(1-s)\Delta\tilde{t}$, while the cost is $c\Delta\tilde{t}$.

Assumption 1 *The gains from trade* $\frac{a}{3}(1-s)-c$ *are positive.*

It will be convenient to capture the gains from trade through the parameter $B = 1 - s - \frac{3c}{a} > 0$ which is positive by assumption 1. Clearly, the gains from trade are smaller the larger the search frictions which are captured by the matching rate a.

Assumption 1 makes it easy to characterize the participation decision of agents in our economy.

Lemma 1 *Agents will stay in the market until they either swap their inventory for their favorite consumption good or are forced to consume. In any equilibrium of the model the expected utility* $R(\tilde{t})$ *from holding a commodity at time* $\tilde{t}$ *is*

$$R(\tilde{t}) = s + B\left(1 - \exp\left(-\frac{a}{3}\left(1-\tilde{t}\right)\right)\right). \tag{2}$$

Proof: We can neglect the probability that an agent who holds a commodity good in her inventory meets an agent who holds fiat money because it is in zero supply. The agent swaps her inventory for her favorite good at rate $\frac{a}{3}$ such that

$$R(\tilde{t}) = \int_0^{1-\tilde{t}} [1-cx]\frac{a}{3}\exp\left(-\frac{a}{3}x\right)dx + \exp\left(-\frac{a}{3}\left(1-\tilde{t}\right)\right)\left(s - c\left(1-\tilde{t}\right)\right)$$

The expected utility is the sum of utilities from obtaining the favorite commodity at time $0 < x < 1-\tilde{t}$ (an event which occurs with probability $\frac{a}{3}\exp\left(-\frac{a}{3}\right)dx$) plus the utility in the event of no favorable match (which occurs with probability $\exp\left(-\frac{a}{3}\left(1-\tilde{t}\right)\right)$. This expression can be simplified and we obtain formula 2. QED

Lemma 1 establishes that all three commodities serve as commodity monies in any equilibrium of the economy. A lot of the literature starting with Kiyotaki and Wright (1989) is concerned with the question of *which* commodities emerge as money. We abstract away from that question because we are mainly interested in the *transition* from commodity to fiat money. By treating all commodities in a symmetric fashion we can keep the underlying commodity money equilibrium as simple as possible.

3 Fiat Money Equilibria

The interesting question is whether our model has equilibria where fiat money is accepted in exchange, e.g. $\tau_i\left(j,0,\tilde{t}\right)=1$ for at least some points in time $\tilde{t}$. We first characterize such potential fiat money equilibria, and then discuss their existence.

We begin by introducing some notation. We denote the share of agents who accept fiat money in equilibrium with $0<\mu\leq 1$, and the rate at which agents meet partners who are prepared to trade their inventory for fiat money with $\lambda=a\mu$. The parameter μ measures the marketability of fiat money. The utility from holding fiat money at time $\tilde{t}$ is $M\left(\tilde{t}\right)$. We can then define the set $N=\left\{\tilde{t}\left|M\left(\tilde{t}\right)=R\left(\tilde{t}\right)\right.\right\}$ which consists of all points in time $\tilde{t}$ when an agent is indifferent between holding fiat money or a commodity good. The supremum of this set is called $\tilde{t}^*$.

Our first result establishes that $\tilde{t}^*$ is well defined.

Lemma 2 *In any fiat money equilibrium* $\tilde{t}^*$ *exists and* $0<\tilde{t}^*<1$.

Proof: The value of holding fiat money decreases to 0 as $\tilde{t} \to 1$ while the value of holding a commodity is bounded below by s. The definition of fiat money equilibrium implies that there exists some $\tilde{t}$ such that $M\left(\tilde{t}\right) \geq R\left(\tilde{t}\right)$. Furthermore, the functions $M\left(\tilde{t}\right)$ and $R\left(\tilde{t}\right)$ are continuous. By the intermediate value theorem the set N is non-empty. Because of continuity there exists some $\epsilon > 0$ such that for each $y \in N$ we have $y < 1 - \epsilon$. QED

We can now conveniently characterize monetary equilibria.

Theorem 1 *In a fiat money equilibrium agents strictly prefer to hold fiat money for all $\tilde{t} \in \left[0, \tilde{t}^*\right)$. They do not accept fiat money in exchange for all $\tilde{t} \in \left(\tilde{t}^*, 1\right]$.*

Proof: See appendix A.

Theorem 1 allows us to express the share of agents μ who accept fiat money in equilibrium in terms of $\tilde{t}^*$.

Corollary 1 *The share of agents who accept fiat money in a fiat money equilibrium is*

$$\mu = \frac{1 - \exp\left(-\frac{a}{3}\tilde{t}^*\right)}{1 - \exp\left(-\frac{a}{3}\right)} < 1. \tag{3}$$

Proof: The share of agents of vintage $\tilde{t}$ who carry a commodity and have not yet matched with an agent who carries their favorite consumption good is $\exp\left(-\frac{a}{3}\tilde{t}\right)$. By assumption all vintages are uniformly distributed and the mass of agents with vintage from the interval $\left(0, \tilde{t}\right)$ who have not yet found a match is

$$\int_0^{\tilde{t}} \exp\left(-\frac{a}{3}x\right) dx = \frac{3}{a}\left[1 - \exp\left(-\frac{a}{3}\tilde{t}\right)\right].$$

Therefore, the mass of searching agents in the economy is $\frac{3}{a}\left[1-\exp\left(-\frac{a}{3}\right)\right]$ and the mass of searching agents who accept fiat money is $\frac{3}{a}\left[1-\exp\left(-\frac{a}{3}\tilde{t}^*\right)\right]$. QED

We can now analyze the conditions under which fiat money equilibria exist. It is quite intuitive that money will not be accepted if search frictions are large (i.e. the matching rate a is small). Agents will not accept fiat money for fear of being unable to exchange it for consumption goods.

Lemma 3 *A sufficient condition such that no fiat money equilibrium exists is:*

$$1-\exp(-a) < s \tag{4}$$

Proof: The probability that a money-holding agent never meets a trading partner is at least $\exp(-a)$. Therefore, the maximum utility of holding fiat money is bounded above by $1\times(1-\exp(-a))$ while the utility of holding a commodity good is bounded below by s. QED

Will fiat money be valuable if search frictions are small? The answer is not obvious because a fiat equilibrium can unravel according to the following feedback mechanism. Assume that all agents accept money such that $\mu_0 = 1$. Because of lemma 2 no individual agent will accept fiat money after some point in time $\tilde{t}_1^* < 1$. In return, the overall marketability of fiat money is reduced to $\mu_1 < \mu_0$. This makes it even less attractive for an individual agent to accept fiat money in exchange and the threshold time after which fiat money is refused decreases to $\tilde{t}_2^* < \tilde{t}_1^*$. By iterating this argument we receive a decreasing sequence μ_n. Fiat equilibria can only exist if this sequence does not converge to 0. Otherwise, no extrinsic beliefs in the marketability of money can be supported.

The next theorem shows that fiat equilibria do indeed exist when search frictions become small. In the limit, fiat money can even become completely marketable.

Theorem 2 *There exists a critical matching rate a^*, such that there exists no fiat money equilibrium for $a < a^*$ and at least one fiat money equilibrium for $a > a^*$ such that the share of agents μ who accept money in exchange is increasing and tends to* 1 *as $a \to \infty$.*

Proof: At time $\tilde{t}^*$ the agent has to be indifferent between accepting money and holding on to her consumption good. Furthermore, agents prefer commodities over fiat money for all $\tilde{t} > \tilde{t}^*$ because of theorem 1. This allows us to characterize $\tilde{t}^*$ as follows:

$$\begin{aligned} M^*\left(a, \tilde{t}^*\right) &= \int_0^{1-t^*} \lambda \exp\left(-\lambda x\right) \left\{\frac{1}{3} + \frac{2}{3} R\left(\tilde{t}^* + x\right)\right\} dx \\ &= s + B\left[1 - \exp\left(-\frac{a}{3}\left(1 - \tilde{t}^*\right)\right)\right] = R^*\left(a, \tilde{t}^*\right) \qquad (5) \end{aligned}$$

Note, that λ depends on both a and $\tilde{t}^*$ (see equation 3), and that this dependence has to be taken into account when solving for $\tilde{t}^*$. The largest solution of this equation (if any exist) defines a fiat equilibrium for the model. It is easy to see that $M^*\left(a, \tilde{t}\right) < R^*\left(a, \tilde{t}\right)$ for $\tilde{t} > \tilde{t}^*$, and $M^*\left(a, \tilde{t}\right) > R^*\left(a, \tilde{t}\right)$ for $\tilde{t} < \tilde{t}^*$ (using a similar argument as in the proof of theorem 1).[2]

Hence we only need to show that (a) equation 5 has indeed a solution for sufficiently large matching rates a, (b) that this solution $\tilde{t}^*$ is increasing in a, and (c) that $\tilde{t}^* \to 1$ as $a \to \infty$.

[2]Note, that this implies that $\frac{\partial M^*}{\partial t} - \frac{\partial R^*}{\partial t} < 0$ in equilibrium.

We can show (a) and (c) by using the following trick. The left hand side of equation 5 captures the gains from holding money if agents follow their optimal strategy. This strategy dominates a simpler strategy where agents hold on to their fiat money until they match up with an agent who holds their favorite consumption good. The gain $\overline{M}\left(a, \tilde{t}\right)$ from this sub-optimal strategy is:

$$\overline{M}\left(a, \tilde{t}\right) = \int_0^{1-\tilde{t}} \frac{\lambda}{3} \exp\left(-\frac{\lambda}{3} x\right) dx \tag{6}$$

The solution $\tilde{t}^{**}$ to the equation $\overline{M}\left(a, \tilde{t}^{**}\right) = R^*\left(a, \tilde{t}^{**}\right)$ satisfies $\tilde{t}^{**} < \tilde{t}^*$. Hence we only have to show that the claim is true for $\tilde{t}^{**}$.

The simplified equation gives the condition:

$$G\left(a, \tilde{t}^{**}\right) = \frac{3c}{a} \tag{7}$$

for $G(a, y) = \exp\left(-\frac{\lambda}{3}(1-y)\right)\left[1 - B \exp\left(-\frac{a}{3}(1-\mu)(1-y)\right)\right]$. We know that $G(a, 1) > \frac{3c}{a}$ for large a. If we fix some arbitrarily small $\epsilon > 0$, we can show that $G(a, 1-\epsilon) < \exp\left(-\frac{1}{6} a\epsilon\right) < \frac{3c}{a}$ for sufficiently small ϵ and sufficiently large a (as $\mu \to 1$ for $\tilde{t}^{**} = 1 - \epsilon$ and $a \to \infty$).

We still have to show that $\tilde{t}^*$ is increasing in a in order to ensure monotonicity of the fiat money equilibrium (part (b) of the proof). We implicitly differentiate the equilibrium condition 5 and find:

$$\frac{d\tilde{t}^*}{da} = -\frac{\frac{\partial M^*}{\partial a} - \frac{\partial R^*}{\partial a}}{\frac{\partial M^*}{\partial t} - \frac{\partial R^*}{\partial t}} \tag{8}$$

We have already shown that $\frac{\partial M^*}{\partial t} - \frac{\partial R^*}{\partial t} < 0$ at equilibrium. It is also not difficult to see that $\frac{\partial M^*}{\partial a} - \frac{\partial R^*}{\partial a} > 0$ in equilibrium. For example, figure 2 plots the difference function $M^*\left(a,\tilde{t}^*\right) - R^*\left(a,\tilde{t}^*\right)$ around the equilibrium $a = 15$ and $\tilde{t}^* = .7966$ ($c = 0.4$, $s = 0.5$). We know that $M^*\left(0,\tilde{t}^*\right) - R^*\left(0,\tilde{t}^*\right) = -s < 0$ and that $M^*\left(\infty,\tilde{t}^*\right) - R^*\left(\infty,\tilde{t}^*\right) = 0$. This implies that the slope of the difference function is positive at the intersection with the a-axis. Therefore $\frac{d\tilde{t}^*}{da}$ is increasing. QED

The results of both lemma 3 and theorem 2 are consistent with our intuition. If search frictions are large only commodity money can exist because it provides insurance to agents who have to consume regularly. By accepting fiat money an agent becomes susceptible to 'hold-up': even if there are agents in the economy who would accept fiat money, she might meet them too late and get stuck with an uneatable good. On the other hand, if search frictions are negligible the hold-up problem is less severe, and a fiat money equilibrium becomes sustainable.

It is noteworthy that the transition from barter to fiat money is not continuous: the equilibrium share of agents who accept money in exchange jumps from 0 to a strictly positive value $\mu^* > 0$ at the critical rate of exchange a^*.

Lemma 4 *The share of agents μ^* who accept fiat money in the unique monetary equilibrium at a^* is strictly greater than zero.*

Proof: Monetary equilibria exist as soon as the difference function $M^*\left(a,\tilde{t}^*\right) - R^*\left(a,\tilde{t}^*\right)$ just touches the $\tilde{t}^*$-axis (see figure 1). Since the difference is strictly less than zero both at $t = 0$ and $t = 1$ we have $0 < \tilde{t}^* < 1$ and $\mu^* > 1$. QED

Numerical Example: A numerical example helps to illustrate how the existence of fiat equilibria depends on the degree of frictions in the economy. We examine an economy where the utility from consuming a non-favorite good is

$s = 0.5$, and the flow cost of participation is $c = 0.4$. We assume that there are gains from trade which implies that $a > 2.4$.

We look at the existence of fiat equilibria by finding solutions to equation 5 as a increases. In figure 1 we plotted the difference function $M\left(\tilde{t}^*\right) - R\left(\tilde{t}^*\right)$ for various values of a. An equilibrium exists if the difference function intersects with the horizontal axis: at these times agents are indifferent between holding money or a commodity good. No fiat equilibria are sustainable for $a < 6.7$. This range is surprisingly large - for $a = 6.7$ agents will eat their favorite consumption good with probability $1 - \exp\left(-\frac{a}{3}\right) = 89.3\%$. Search frictions therefore do not have to be particularly large for fiat equilibria to break down. For $a > 6.7$ there is an equilibrium based on the largest solution $\tilde{t}^*$ to the difference equation. Furthermore, the share of agents who accept fiat money tends to 1 as a increases as figure 3 shows.

Fiat equilibria in the region $a \in (2.4, 6.7)$ unravel because beliefs in the marketability of fiat money become unsustainable. For example, assume $a = 3$ and all agents are willing to accept fiat money. In this case it is optimal for an individual agent to accept money up to time $\tilde{t} = 0.48$. This decreases the overall acceptability of money to 60.4 percent which is no longer enough to sustain a fiat equilibrium.

4 Conclusion

In the basic Kiyotaki-Wright framework agents accept an object other than their consumption good (commodity or fiat money) because it is more marketable or has better intrinsic characteristics (storability, durability etc.) than the object they currently hold. The intrinsic value of different media of exchange is of no concern to agents because all of them have zero consumption value.

In this paper we argued that this specification neglects a crucial difference between a physical object and paper currency: while the latter is intrinsically worthless for every agent, the former might well have consumption value to an agent even if it is not her favorite good. Its superior intrinsic value can make commodity money a safer medium of exchange than fiat currency despite its otherwise inferior intrinsic characteristics, because it ensures an agent against not finding a suitable trading partner. If search frictions are small this insurance motive for holding commodity money is negligible. However, under large search frictions this type of hold-up becomes a concern.

Our model thus gives us a natural way to think about the transition between monetary regimes in terms of the degree of friction in the economy. In developing economies (large frictions) fiat money cannot be supported by any extrinsic beliefs and therefore cannot arise as a medium of exchange. In advanced economies with better market mechanisms (small frictions) fiat equilibria can be supported.

We have deliberately kept our model as simple as possible and only looked at the injection of fiat money at the margin. It would be interesting to generalize our results to larger infusions of fiat currency. We would also like to analyze more general fiat currencies. For example, Kiyotaki and Wright (1991) showed in a standard framework that fiat money can have value even if it has inferior intrinsic properties. We expect that it would be considerably harder to generate this phenomenon in our framework.

A Proof of Theorem 1

We look at some $y \in N$ and compare the value of holding fiat money at some time $y-\epsilon$ for some small ϵ with the value of holding a commodity. More specifically, we look at utility $\tilde{M}(y-\epsilon)$ where the agent only exchanges her unit of fiat money for her favorite consumption good for $\tilde{t} \in (y-\epsilon, y)$ and follows her optimal strategy from then on. Clearly, $M(y-\epsilon) \geq \tilde{M}(y-\epsilon)$.

We can calculate $\tilde{M}(y-\epsilon)$ as follows. In the time interval $\tilde{t} \in (y-\epsilon, y)$ an agent who holds fiat money meets another agent who both carries her favorite consumption good and is prepared to exchange it for money at rate $\frac{\lambda}{3}$. Therefore, the agent obtains her favorite consumption good during that time with probability $1-\exp\left(-\frac{\lambda}{3}\epsilon\right)$ and otherwise follows her optimal strategy afterwards which gives utility $M(y) = R(y)$ by assumption. Hence,

$$\tilde{M}(y-\epsilon) = 1 - \exp\left(-\frac{\lambda}{3}\epsilon\right) + \exp\left(-\frac{\lambda}{3}\epsilon\right) R(y). \tag{9}$$

We can then calculate:

$$\tilde{M}(y-\epsilon) - R(y-\epsilon) = \epsilon\left[c\mu - \frac{B}{3}(1-\mu)\, a \exp\left(-\frac{a}{3}(1-y)\right)\right] + O\left(\epsilon^2\right) \tag{10}$$

Money has to be valuable within a small interval $\left(\tilde{t}^* - \epsilon, \tilde{t}^*\right)$ due to the way we defined $\tilde{t}^*$. Therefore, $\tilde{M}\left(\tilde{t}^* - \epsilon\right) = M\left(\tilde{t}^* - \epsilon\right)$ and

$$D\left(\tilde{t}^*\right) = c\mu - \frac{B}{3}(1-\mu)\, a \exp\left(-\frac{a}{3}\left(1-\tilde{t}^*\right)\right) > 0. \tag{11}$$

We can now prove by contradiction that there cannot be some $\tilde{t} < \tilde{t}^*$ such that $M\left(\tilde{t}\right) < R(t)$. In this case there would exist $y \in N$ such that $M(y-\epsilon) - R(y-\epsilon) < 0$. However, we know that

$$M(y-\epsilon) - R(y-\epsilon) > \tilde{M}(y-\epsilon) - R(y-\epsilon) = D(y)\,\epsilon + O\left(\epsilon^2\right) > 0, \qquad (12)$$

because $D(y) > D\left(\tilde{t}^*\right)$ as $y < \tilde{t}^*$. QED

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Figure 1: Difference function $M^* \left(a, \tilde{t}^*\right) - R^* \left(a, \tilde{t}^*\right)$ for fixed $a = 3$ (solid line), $a = 6.7$ (dashed line) and $a = 15$ (dotted line) ($c = 0.4$, $s = 0.5$)

Figure 2: Difference function $M^*\left(a,\tilde{t}^*\right) - R^*\left(a,\tilde{t}^*\right)$ for fixed $\tilde{t}^* = .7966$ ($c = 0.4$, $s = 0.5$)

Figure 3: Acceptance rate μ of fiat money in fiat money equilibrium ($c = 0.4$, $s = 0.5$)

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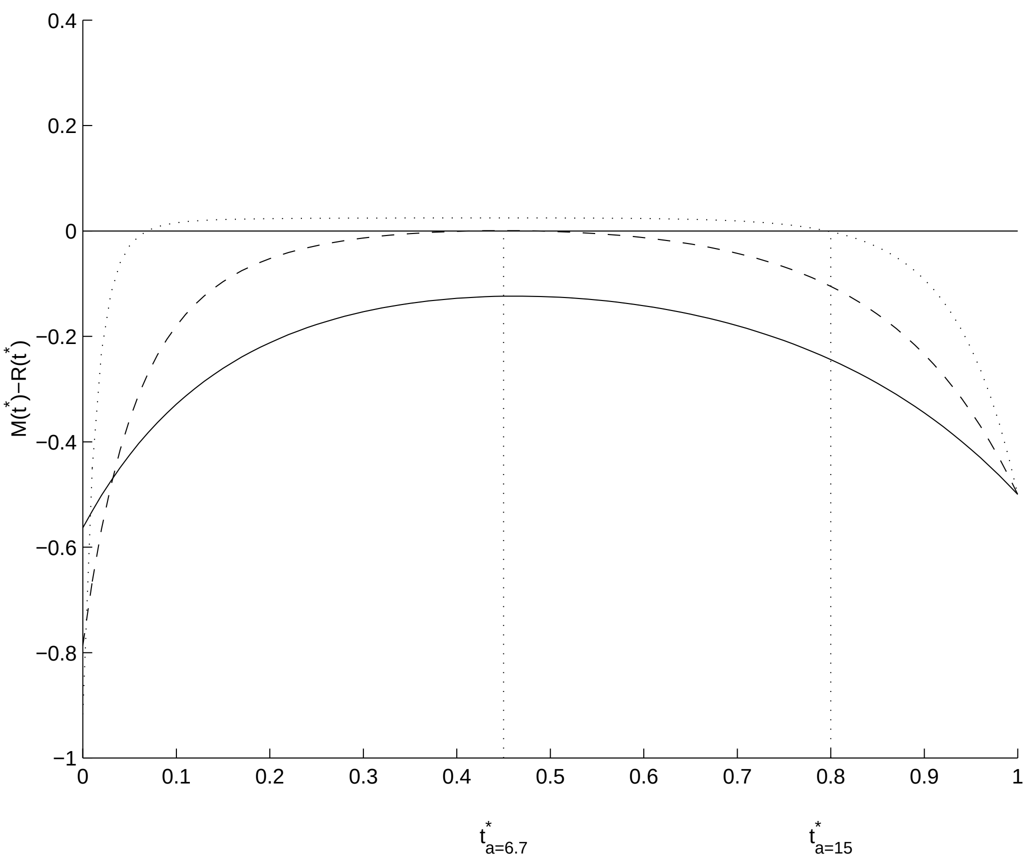

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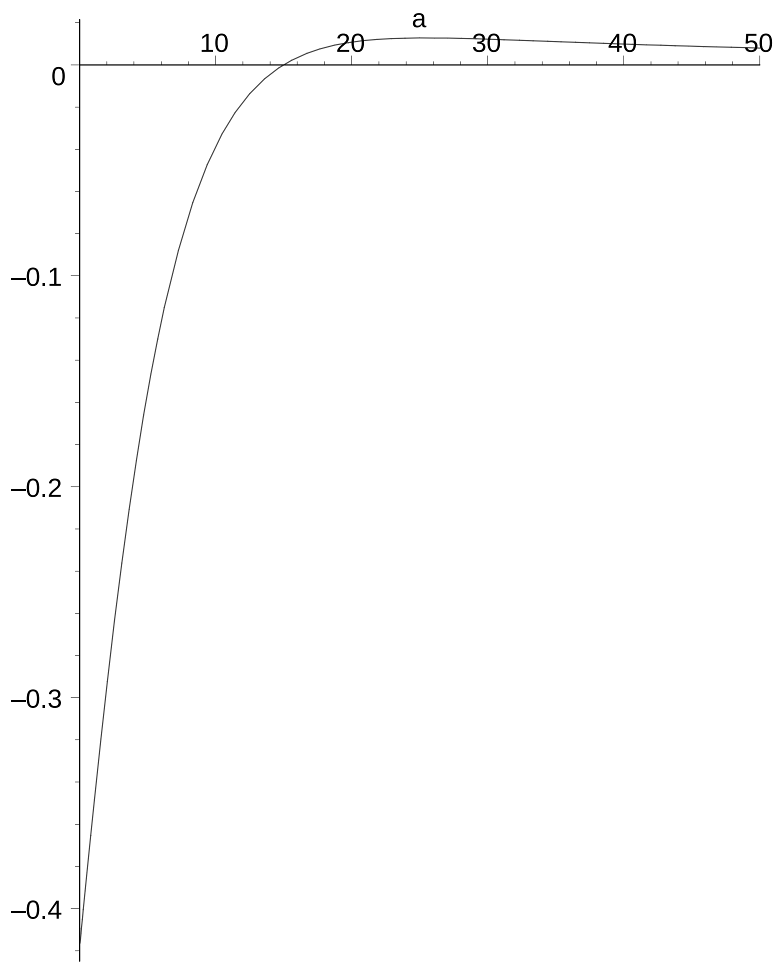

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